Spatial Phenomena Exhibiting Extremes

Modelling Extremes with the Spatial Vine Copula

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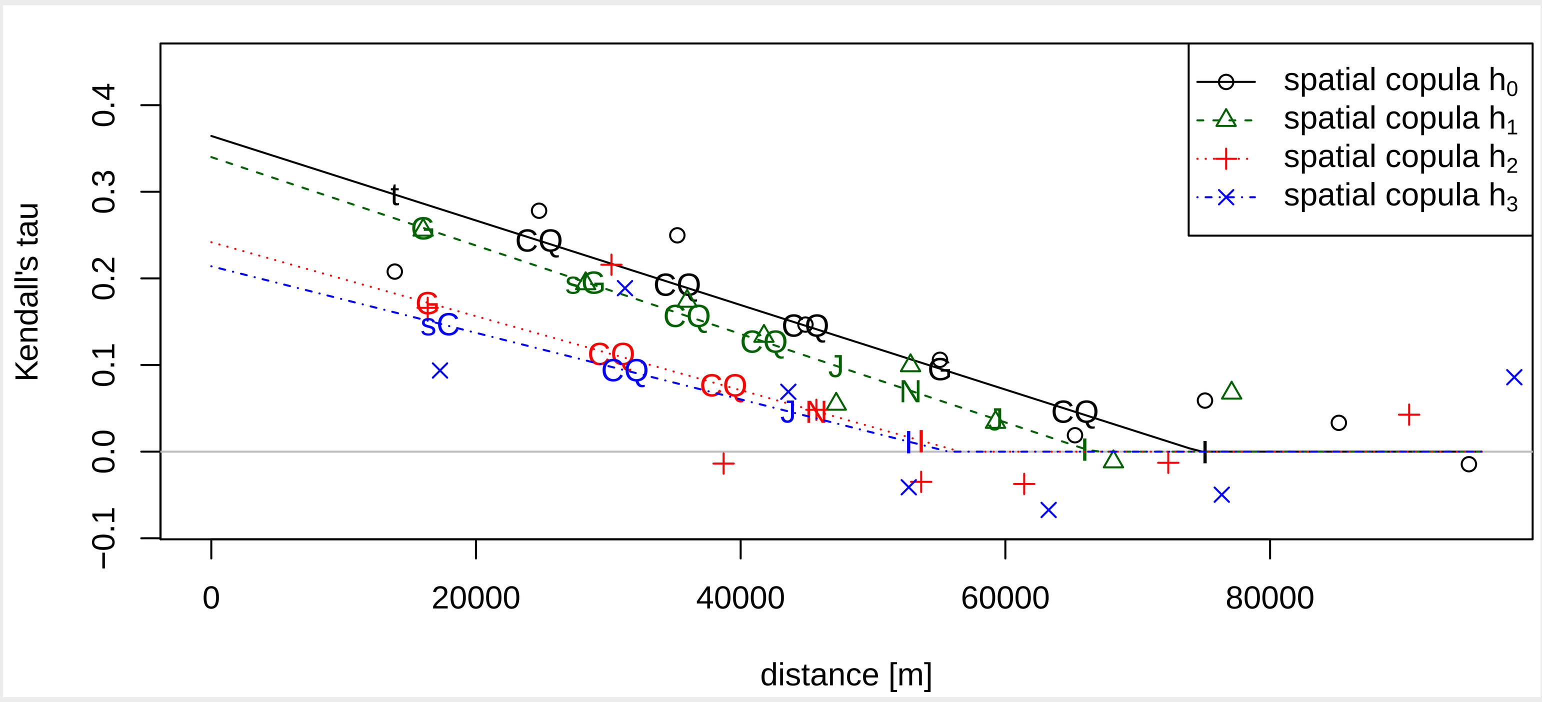
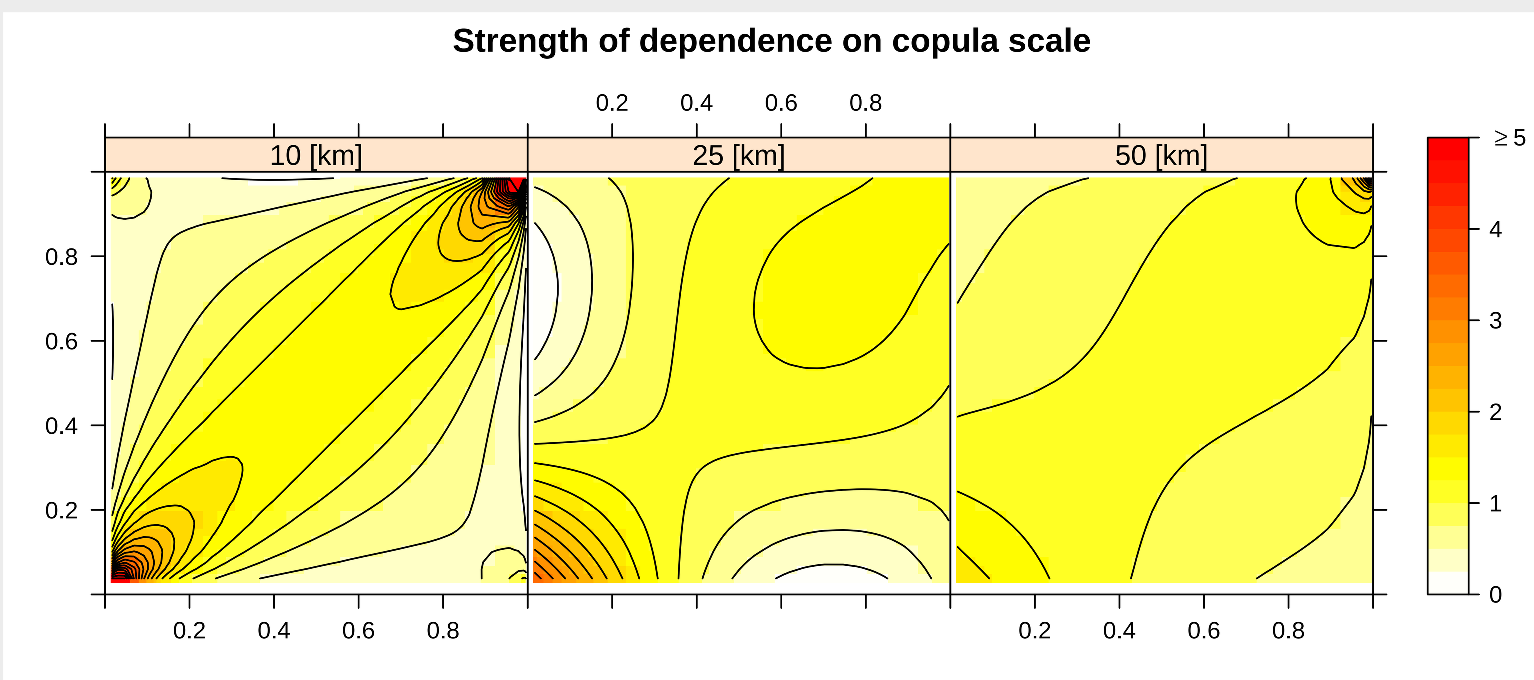
Definition of the Spatial Vine Copula

Spatial dependence between locations may change with distance in *strength*:



The four bivariate spatial copulas used in the 5-dim. spatial vine copula. The copula families are denoted by: Gaussian "N", student "t", Clayton "C", Frank "F", Gumbel "G", Joe "J", survival Clayton "sC", survival Gumbel "sG", cubic-quadratic Sec. "CQ", product "I").

and *shape* between the CDFs of point pairs:



Structural changes of the strength of dependence with distance.

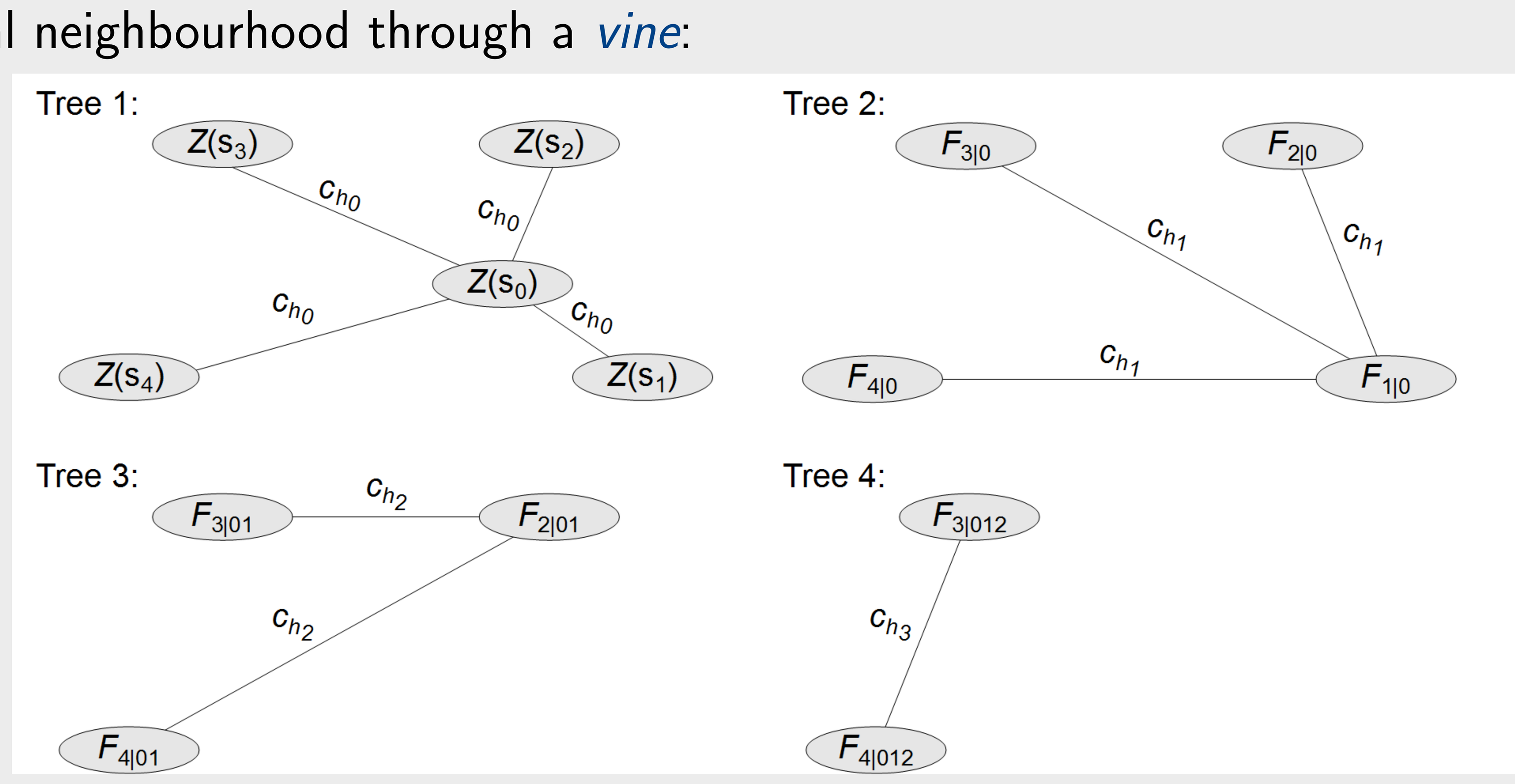
Asymmetric dependence structures (i.e. non-Gaussian) may be present.

Spatial copulas can represent dependence structures that change with distance. Their density is given as convex combination of bivariate copulas:

$$c_h(u,v) := \begin{cases} c_{1,h}(u,v) & , 0 \le h < l_1 \\ (1-\lambda_2)c_{1,h}(u,v) + \lambda_2 c_{2,h}(u,v) & , l_1 \le h < l_2 \\ \vdots & \vdots \\ (1-\lambda_k)c_{k-1,h}(u,v) + \lambda_k \cdot 1 & , l_{k-1} \le h < l_k \\ 1 & , l_k \le h \end{cases}$$

where $\lambda_j := \frac{h-l_{j-1}}{l_j - l_{j-1}}$, h denotes the separating distance and $l_1, \dots, l_k$ denote the representative distances of the bins (e.g. mean distance of all involved point pairs).

Spatial vine copulas join pair-wise *spatial copulas* into a multivariate distribution of a local neighbourhood through a *vine*:



Graphical representation of a spatial vine copula. Each tree i has its own bivariate spatial copula $c_{h_{i-1}}$ describing the changing dependence between pairs of locations.

Its density is given through the product of all involved bivariate copula densities:

$$c_{\mathbf{h}}(u_0,\dots,u_d) = \prod_{i=1}^{d} c_{h_0(i)}(u_0,u_i) \cdot \prod_{j=1}^{d-1}\prod_{i=1}^{d-j} c_{h_j(j+i)}(u_{j|0,\dots,j-1}, u_{j+i|0,\dots,j-1})$$

where $u_i = F_i(Z(s_i))$ for $0 \le i \le d$ and

$$u_{j+i|0,\dots,j-1} = F_{h_{j-1}(j+i)}(u_{j+i}|u_0,\dots,u_{j-1}) = \frac{\partial C_{h_{j-1}(j+i)}(u_{j-1|0,\dots,j-2}, u_{j+i|0,\dots,j-2})}{\partial u_{j-1|0,\dots,j-2}}$$

A full distribution is obtained by multiplying the spatial vine copula with the marginal densities.

Usage of the Spatial Vine Copula

Predictions are obtained from the full distribution by means of any p-quantile (i.e. the *median*) or the *expected value*

$$\hat{x}_p = F^{-1}\left(C^{-1}(p|u_1,\dots,u_d)\right) \qquad \hat{x}_m = \int_{[0,1]} F^{-1}(u)\cdot c(u|u_1,\dots,u_d)\,\mathrm{d}u.$$

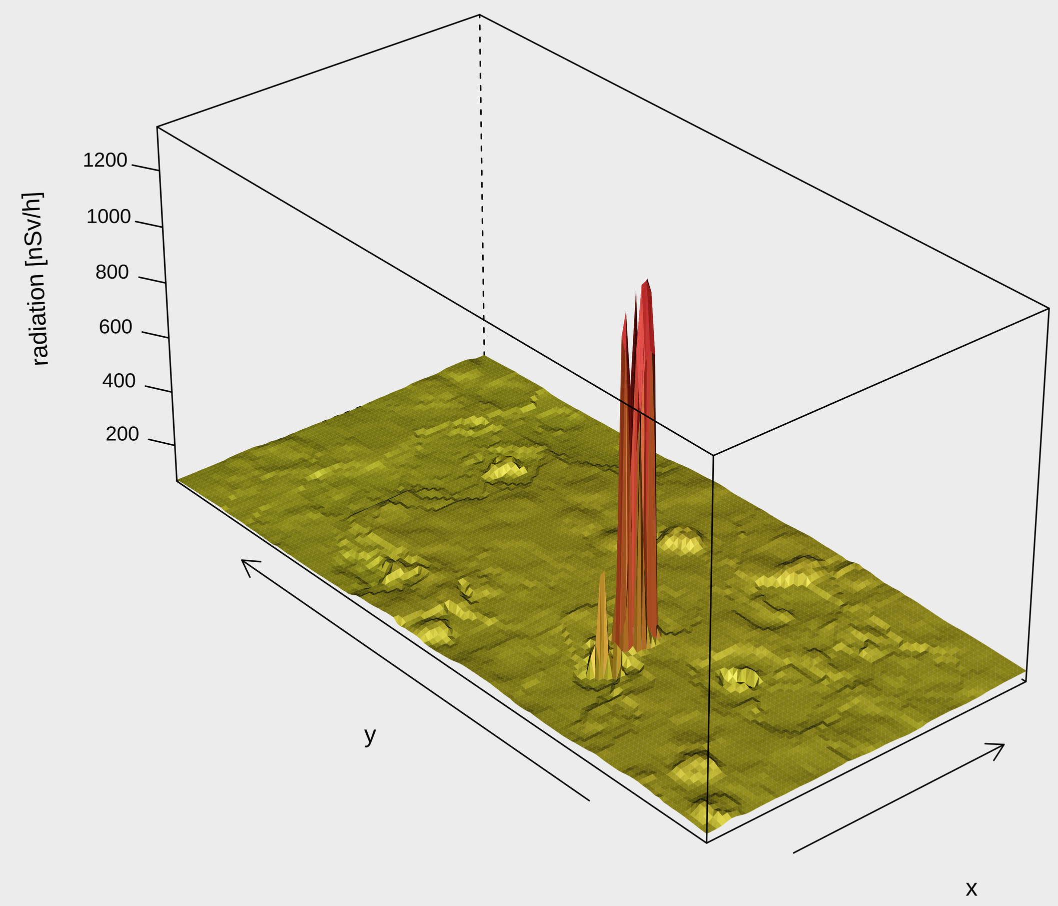
Simulations can be drawn from the conditional density at each location following a random path.

Uncertainties may follow any distribution and are given through a full but family free conditional distribution.

Application of the Spatial Vine Copula

Use case: Interpolation of the "Joker" dataset from the Spatial Interpolation Comparison 2004: simulated radiation including an accidental release of radioactive material.

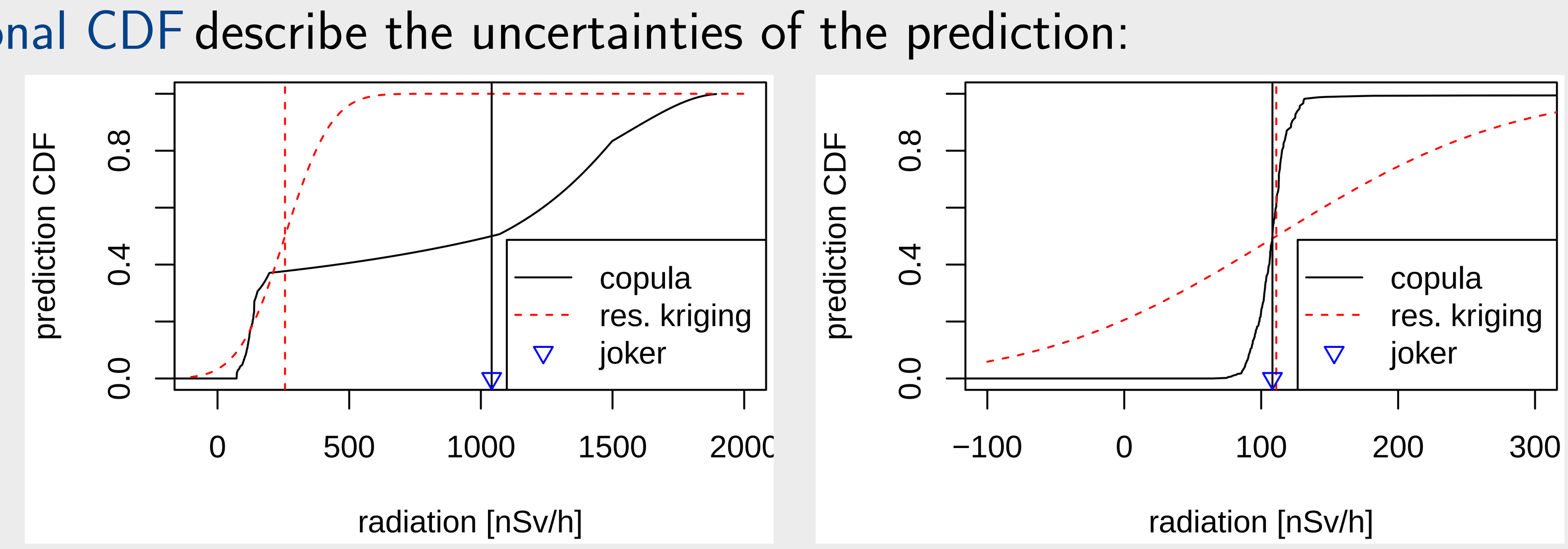
Interpolation results for the median spatial vine copula predictor:



Validation: 808 additionally simulated locations are used to validate the interpolation based on 200 locations.

approach	MAE	RMSE	ME	COR
spatial vine copula	*14.5*	*67.6*	-6.1	*0.60*
TG log-kriging	20.8	78.2	*-2.1*	0.39
residual kriging	21.1	75.6	5.2	0.43

Conditional CDF describe the uncertainties of the prediction:



Conditional CDFs of the median spatial vine copula predictor and residual kriging at an extreme (left) and a background (right) location.

Software and code is available as R-package *spcopula* on r-forge (talk *S2.2* on Thursday).

Conclusions

The spatial vine copula ...

- *flexibly describes spatial dependence* of local neighbourhoods.
- is able to *capture extremes*.
- allows to use *any marginal distribution*.
- *outperforms* "classical" geostatistical approaches (in terms of MAE even the best SIC2004 participant).
- directly provides *conditional distributions describing uncertainties*.

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