Using vine copulas to explore relationships between variables determining deforestation in the Brazilian Amazon

GIScience for environmental change - use cases in the Brazilian Amazon
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Deforestation

- the process of deforestation is complex
- the size of the area is hard to grasp
- the number of potential explanatory variables is large

Probabilistic speaking:
We need to capture/model/understand a multi-variate random process over a large spatial/spatio-temporal region.

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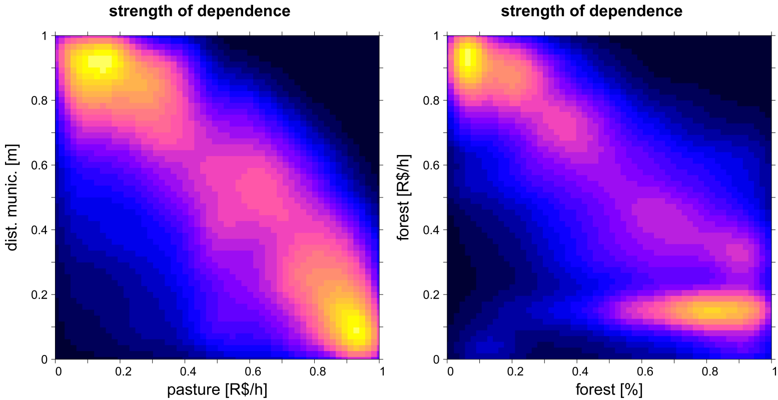
Additional challenges

- marginal distributions are of various kinds (i.e. non-Gaussian)
- dependence structures are asymmetric (i.e. non-elliptical)
- some variables exhibit joint tail-dependence

Thus, the assumption of an underlying Gaussian process is questionable.

smoothed scatter plots showing strength of dependece



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Vine Copulas

- the theory of *vine copulas*[1] allows to break up a multi-variate distribution into a set of *bivariate* ones
- any bivariate one can be chosen freely
- margins and dependence structures (represented by *copulas*) can be handled independently

[1] see [1] for further details

C-vines

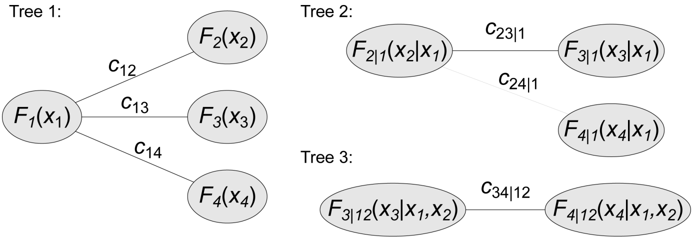

Figure: a 4-dimensional canonical vine

$$f_C(x_1,\dots,x_4) = \prod_{k=1}^{4} f(x_k) \prod_{j=1}^{4-1} \prod_{i=1}^{4-j} c_{j,j+i|1\dots j-1}(\cdots)$$

D-vines

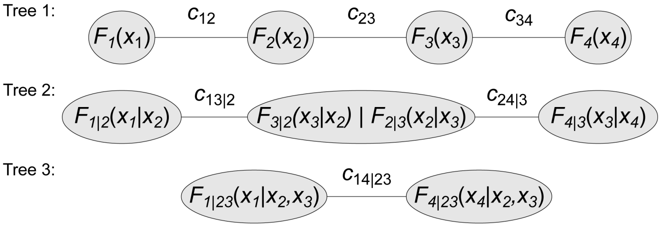

Figure: a 4-dimensional D-vine

$$f_D(x_1,\dots,x_4) = \prod_{k=1}^{4} f(x_k) \prod_{j=1}^{4-1} \prod_{i=1}^{4-j} c_{i,i+j|i+1\dots i+j-1}(\cdots)$$

R-vines

- lie "in between" C and D-vines
- are by far more flexible

One has to choose

- there are many different ways to decompose a multivariate distribution into bivariate ones
- typically, different choices will give different results

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Maximum Spanning Trees

Definition

Given a connected, undirected and weighted graph. A *maximum spanning tree* is a cycle-free subgraph that connects all vertices and its sum over all edges is larger or equal than for any other such cycle-free subgraph.

Maximum spanning trees and R-vines

We derive a maximum spanning tree by (following [2]):

1. calculating Kendall's tau for all pairs of variables of interest
2. building a fully connected and undirected graph over all these variables
3. assigning Kendall's tau as weights for the corresponding edges
4. calculating the maximum spanning tree for this graph

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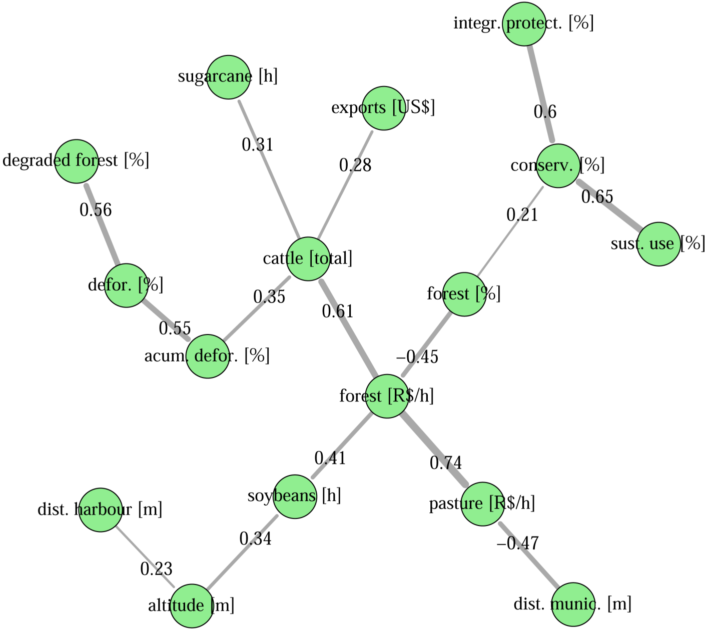
A maximum spanning tree over deforestation variables



References

- Aas, Kjersti, Claudia Czado, Arnoldo Frigessi & Henrik Bakken (2009). Pair-copula constructions of multiple dependence. *Insurance: Mathematics and Economics, 44*, 182 - 198.
- Dißmann, J., E. C. Brechmann, C. Czado, and D. Kurowicka (2011). Selecting and estimating regular vine copulae and application to financial returns. *Submitted for publication.*

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