Spatial Phenomena and Joint Return Periods in R

Modelling Spatial Phenomena and Joint Return Periods with Copulas using R: the spcopula Package

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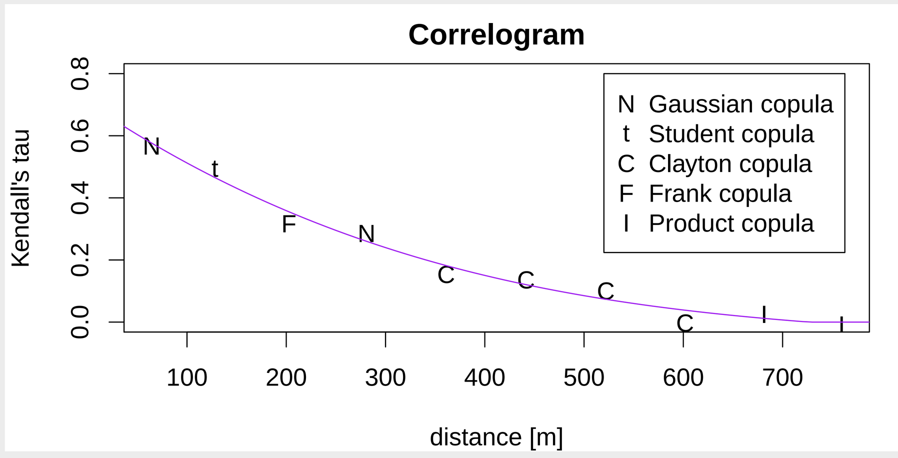
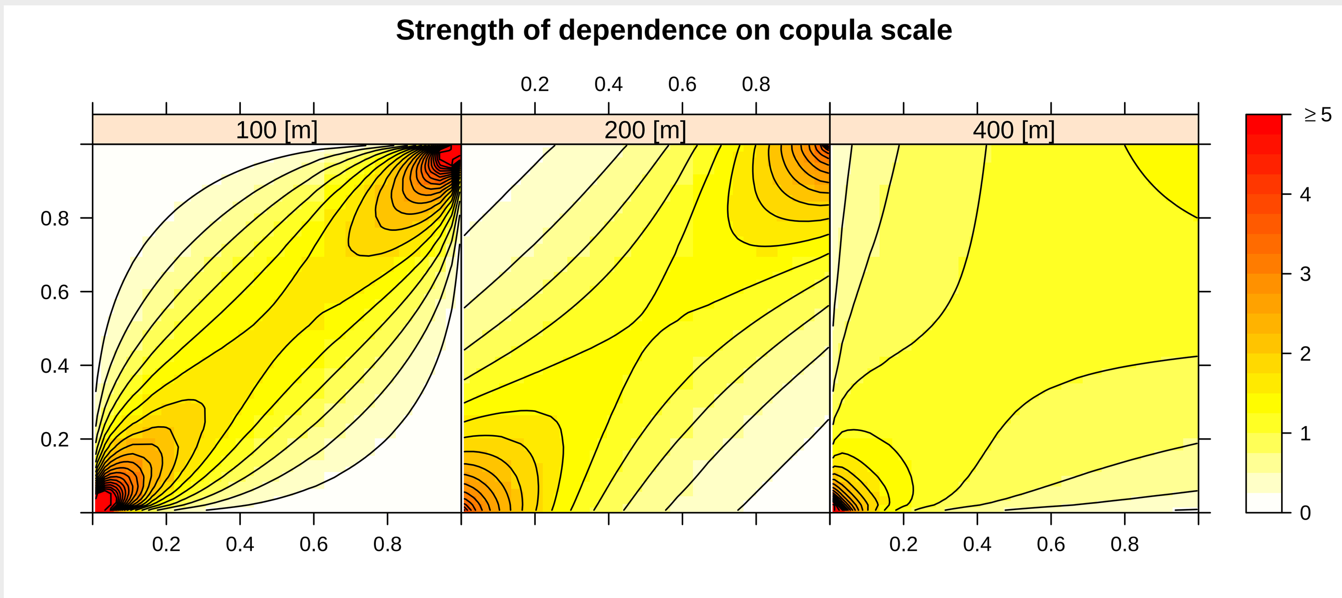
Spatial Phenomena

Spatial phenomena are assumed to be stationary isotropic random fields.

Spatial dependence may change with distance in *strength*:



and *shape* between the CDFs of point pairs:



Asymmetric dependence structures (i.e. non-Gaussian) might be present.

Spatial copulas can represent dependence structures that change with distance.

Spatial vine copulas join pair-wise *spatial copulas* into a multivariate distribution of a local neighborhood through a vine copula.

Predictions are obtained from the full distribution by means of any p-quantile (i.e. the *median*) or the *expected value*

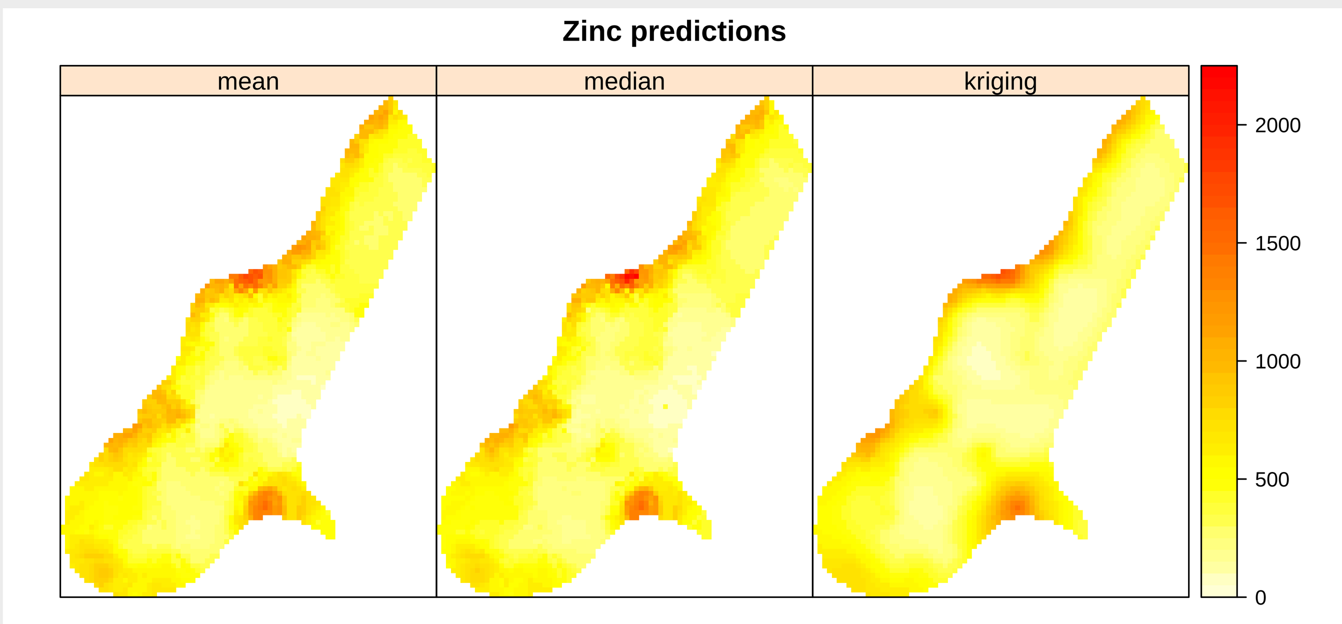
$$\hat{x}_p = F^{-1}\left(C^{-1}(p|u_1,\dots,u_d)\right) \qquad \hat{x}_m = \int_{[0,1]} F^{-1}(u)\cdot c\left(u|u_1,\dots,u_d\right)\mathrm{d}u.$$

Application: Interpolation of zinc concentrations along the Meuse river bank.

R code:

```
bins <- calcBins(meuse,var="zinc",nbins=10,cutoff=800)
calcKTauPol <- fitCorFun(bins, degree=3)
spCop <- spCopula(components=list(...),
                  distances=bins$meanDists,
                  spDepFun=calcKTauPol, unit="m")
meuseNeigh <- getNeighbours(meuse, var="zinc", size=5)
meuseSpVine <- fitCopula(spVineCopula(spCop, vineCopula(4L)),
                         data=meuseNeigh)
```

Interpolation results for mean and median spatial vine copula predictors compared with trans-Gaussian kriging using the log-transform:



The spcopula package ...

uses the *VineCopula* package

extends the *copula* package

builds on the packages *sp* and *spacetime* for data handling

provides flexible *bivariate spatial copulas* being the building blocks of *spatial* and *spatio-temporal vine copulas*

allows to interpolate, derive confidence bands and to calculate exceedance probabilities

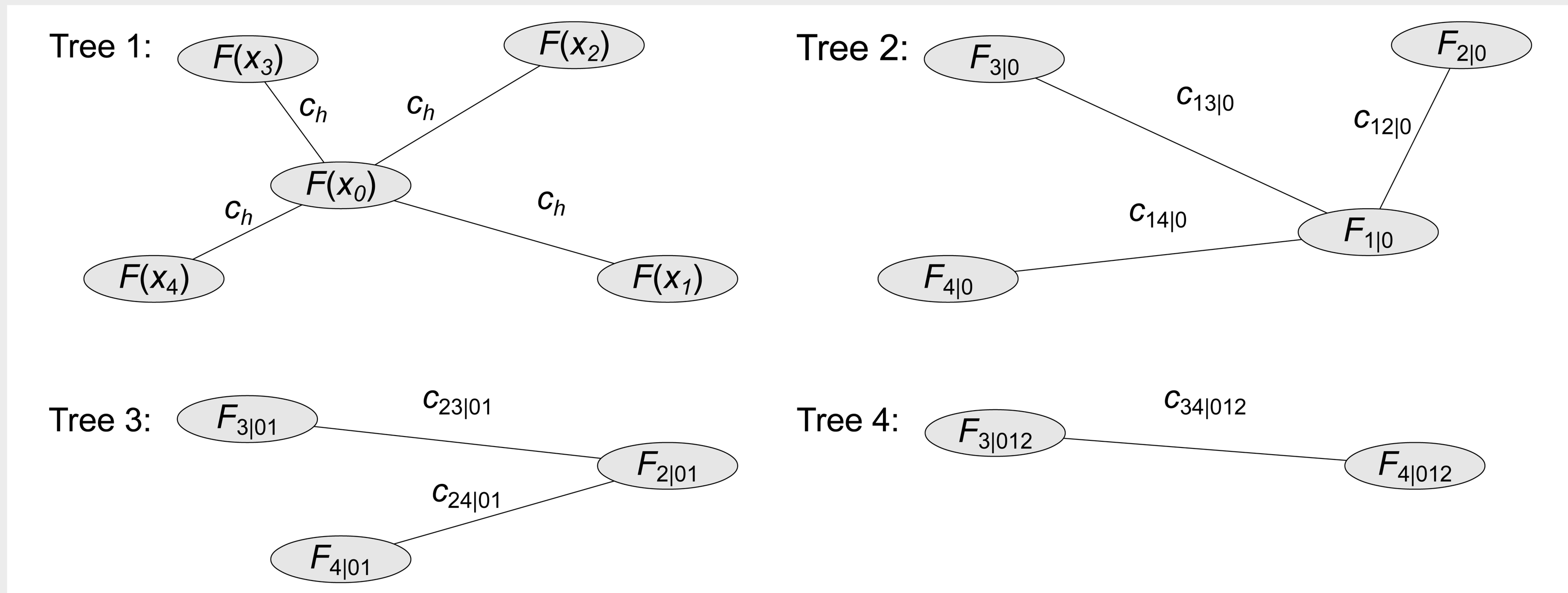
offers functions to calculate design events for *joint return periods* of the *OR* and *Kendall type*

is available on r-forge

Vine Copulas ...

- approximate multivariate copulas, modeling *multivariate dependence structures*.
- of dimension d rely on $1/2(d-1)d$ bivariate copulas.
- allow to mix different copula families without limitations and are thus *very flexible*.
- iteratively re-use well established estimation procedures for bivariate copulas.
- combine independently fitted marginal distributions to a *full multivariate distribution* of the observed phenomenon.
- allow to derive conditional densities from their multivariate density.

Graphical representation of a 5-D (spatial) canonical vine copula



A *vine copula density* is the product of all involved bivariate copula densities; in the 3-D case:

$$c(u,v,w) = c_{UW|V}\left(F_{U|V}(u|v), F_{W|V}(w|v)\right)\cdot c_{UV}(u,v)\cdot c_{VW}(v,w)$$

where

$$F_{U|V}(u|v) = \frac{\partial C_{UV}(u,v)}{\partial v} \quad \text{and} \quad F_{W|V}(w|v) = \frac{\partial C_{VW}(v,w)}{\partial v}$$

and $u = F_X(x)$, $v = F_Y(y)$ and $w = F_Z(z)$ for some marginal cumulative distribution functions.

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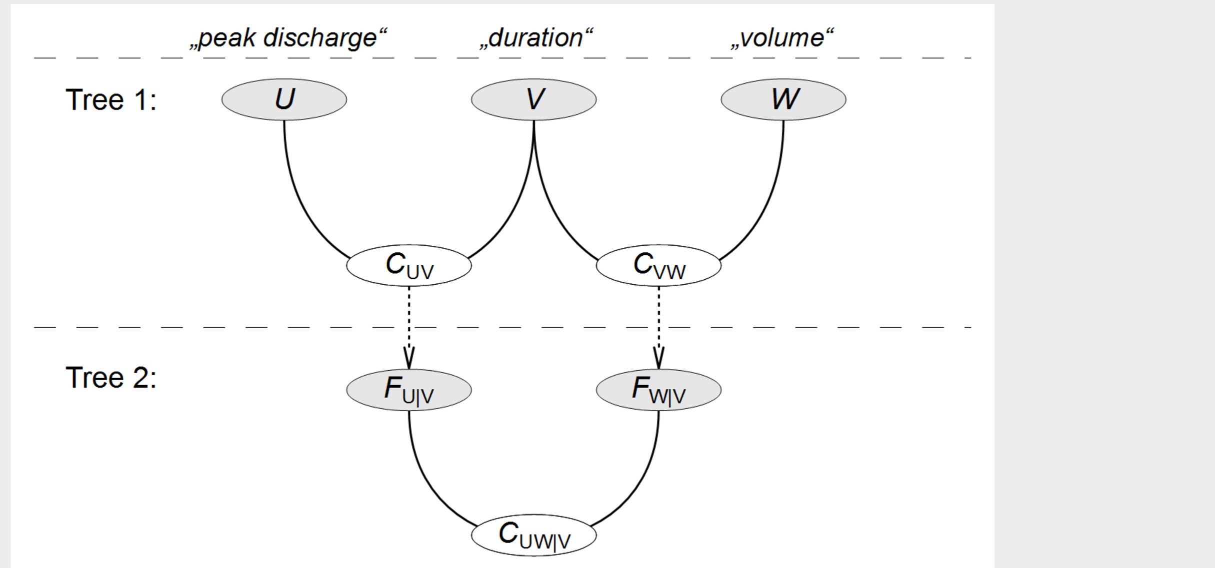
Joint Return Periods

Return periods describe the inter-arrival time T of extreme events usually identifying the $1/T$ harmful events.

Multivariate extremes (e.g. peak discharge, duration and volume) are likely to be strongly correlated and lack a unique natural ordering.

Vine copulas can be used to build multivariate distributions describing extreme events yielding (ensembles of) design events:



Multivariate densities are given through the product

$$f(q_p,d,v_p) = c\left(F_{Q_p}(q_p), F_D(d), F_{V_p}(v_p)\right)\cdot f_{Q_{max}}(q_p)\cdot f_D(d)\cdot f_{V_p}(v_p)$$

where F_{Q_p}, F_D, F_{V_p} and f_{Q_p}, f_D, f_{V_p} are the marginal CDFs and PDFs.

Kendall Return Periods can be used to identify events defining a *critical layer*, which is defined through the cumulative distribution function as in the univariate approach identifying a *unique sub-critical region*.

Application: 500 simulated triples (Q_p, D, V_p) of annual flood maxima

Log-likelihoods: 3-dimensional Gaussian copula: 935; vine copula: 1047

Joint return periods are derived through simulation and numerical integration.

R code:

```
vineCop <- fitCopula(vineCopula(3L,"DVine"), data=rtTriples)
empVine <- genEmpCop(vine3d, sample.size=1e5)
kenVine <- genEmpKenFun(empVine, sample=empVine@sample)

kendallRP(kenVine, cl=0.9) # more than 40 years
criticalLevel(kenVine, KRP=10) # Kendall critical level: 0.730
```

Critical design events (here for $T_{\mathrm{KEN}} = T_{\mathrm{OR}} = 10yr$) can be found by selecting the *most likely critical design event* for

the *OR type*

$$T_{\mathrm{OR}} = \frac{\mu_T}{1-t_{\mathrm{OR}}}$$
$$\Leftrightarrow t_{\mathrm{OR}} = 1-\frac{\mu_T}{T_{\mathrm{OR}}}$$
$$t_{\mathrm{OR}} = 0.9$$

$$(u_{\mathrm{OR}}, v_{\mathrm{OR}}, w_{\mathrm{OR}}) = \underset{C(u,v,w)=t_{\mathrm{OR}}}{\operatorname{argmax}} f\left(F_{Q_p}^{-1}(u), F_D^{-1}(v), F_{V_p}^{-1}(w)\right) \approx (0.948, 0.936, 0.960)$$

and the *Kendall type*

$$T_{\mathrm{KEN}} = \frac{\mu_T}{1-K_C(t_{\mathrm{KEN}})}$$
$$\Leftrightarrow t_{\mathrm{KEN}} = K_C^{-1}\left(1-\frac{\mu_T}{T_{\mathrm{KEN}}}\right)$$
$$t_{\mathrm{KEN}} \approx 0.730$$

$$(u_{\mathrm{KEN}}, v_{\mathrm{KEN}}, w_{\mathrm{KEN}}) = \underset{C(u,v,w)=t_{\mathrm{KEN}}}{\operatorname{argmax}} f\left(F_{Q_p}^{-1}(u), F_D^{-1}(v), F_{V_p}^{-1}(w)\right) \approx (0.844, 0.820, 0.851)$$

and transformation through the marginal CDFs

$$(q_p, d, v_p)_{\mathrm{OR}} \approx \left(213m^3/s, 18.41h, 3.1\ 10^6m^3\right)$$

$$(q_p, d, v_p)_{\mathrm{KEN}} \approx \left(147m^3/s, 12.90h, 1.8\ 10^6m^3\right)$$

For a comparison study on *uni- and multivariate return period approaches* see:

Gräler et al.: *Multivariate return periods in hydrology: a critical and practical review focusing on synthetic design hydrograph estimation*. Hydrol. Earth Syst. Sci., 17, 1281-1296, 2013.