Chapter 5
Amazon copula

Seminar *Spatio-temporal dependence*,
07.02.2011 - 11.02.2011

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Outline

1. **a few strange copulas**
   - asymmetric copulas
   - copulas for zero inflated data

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An asymmetric copula - ASC I

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So far, all the families we looked at were symmetric copula families. A two parameter asymmetric copula is given as follows:

Example

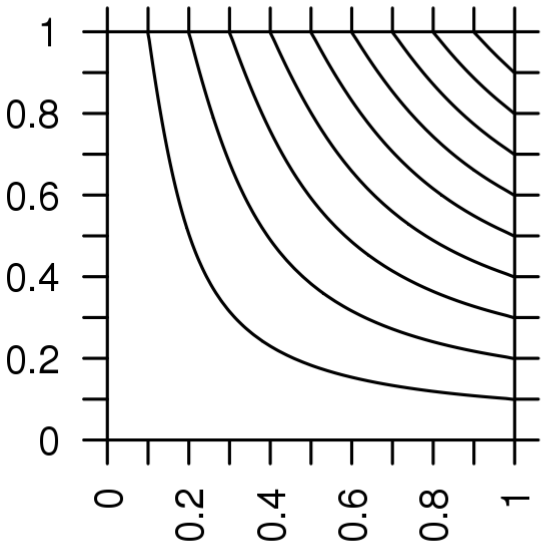
A family of copulas which is not symmetric is the following

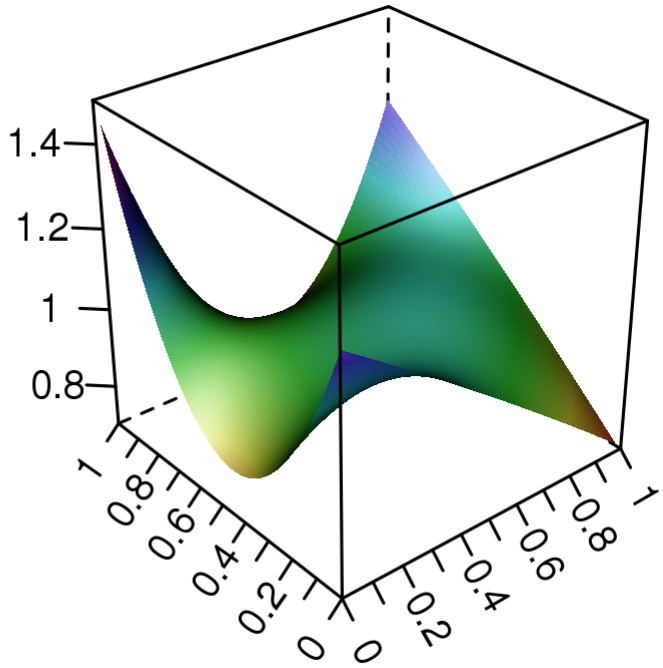
$$C^A_{ab}(u,v) = uv + uv(1-u)(1-v)\big((a-b)v(1-u)+b\big)$$

for all $|b| \leq 1$ and $\big(b-3-\sqrt{9+6b-3b^2}\big)/2 \leq a \leq 1$ with $a \neq b$ (see Example 3.16 in [Nelsen 2006]). We will denote this two parameter family as family of asymmetric copulas (ASC).

An asymmetric copula - ASC II

Contourplot and density for one pair of parameters are illustrated below:





An asymmetric copula - ASC III

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An asymmetric copula can be estimated using a combination of the inversion of Kendall's tau/Spearman's rho and a maximum likelihood estimation [Gräler 2009]. Where the parameter a can be estimated by:

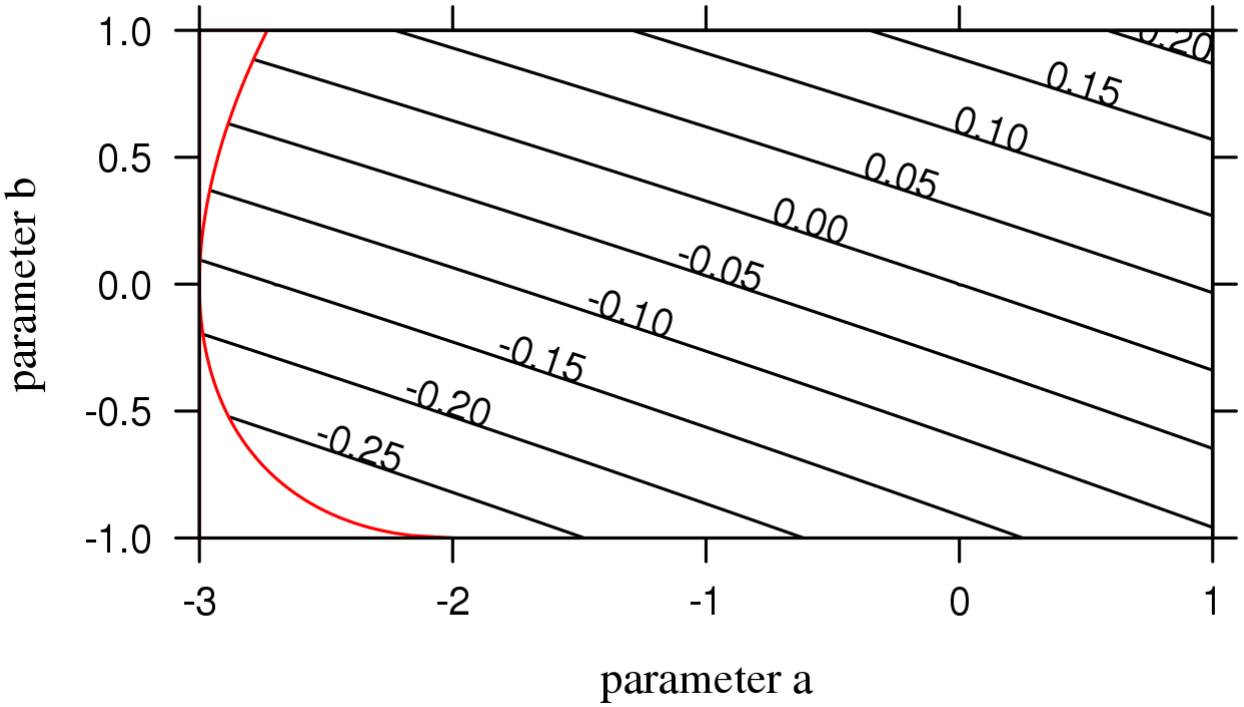
$$\hat{a} = \frac{450\tau - 75b + b^2}{25 - b}$$

or

$$\hat{a} = 12\rho - 3b$$

The parameter b than needs to be fitted using a maximum likelihood estimator. Valid parameters can only be obtained within the following region:

An asymmetric copula - ASC IV



Where the contour lines denote the value of Kendall's tau.

Zero inflated data I

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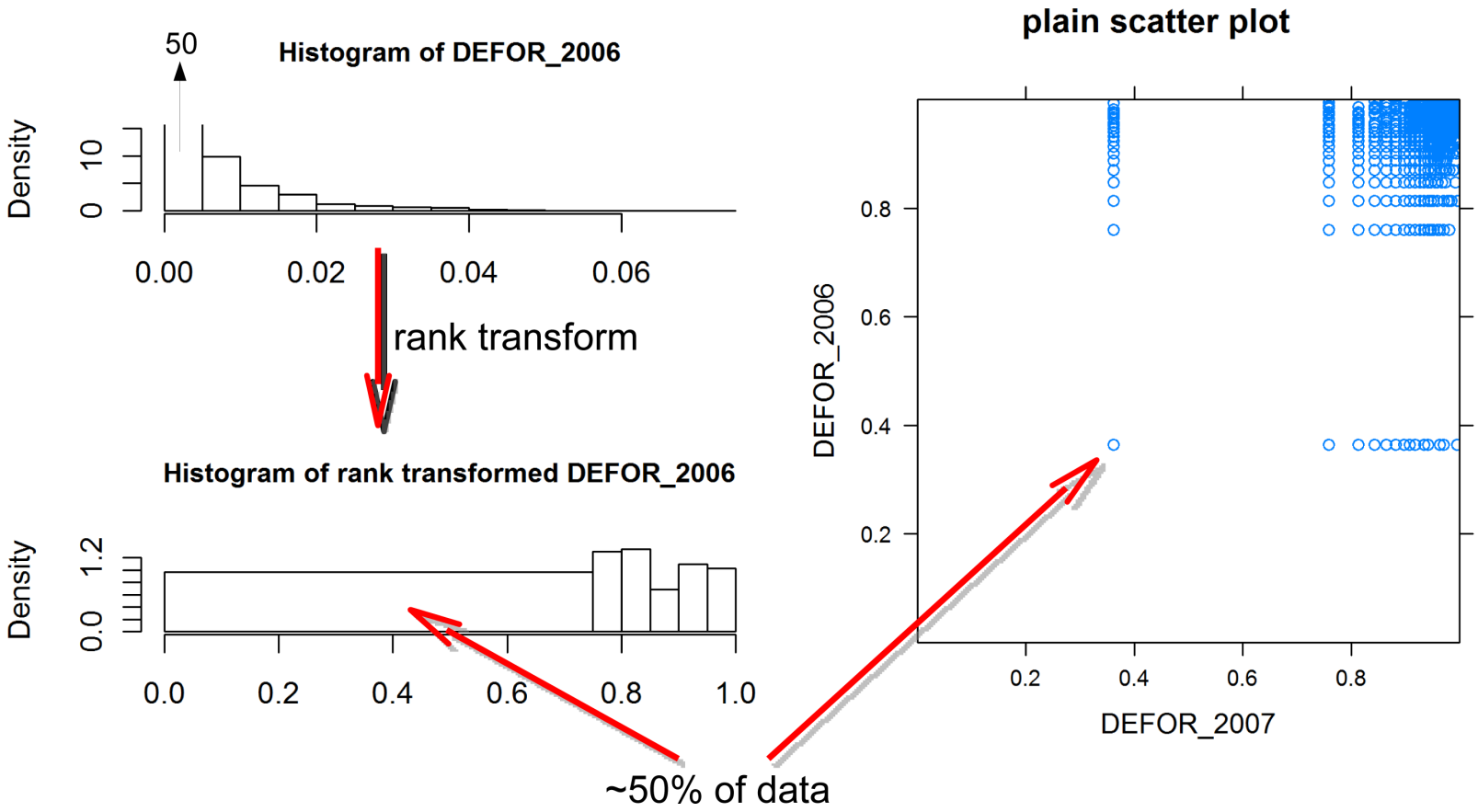
In several applications one will find a huge amount of 0s (or very small values) in a sample. This is the case for example for

- rainfall data
- nuclear radiation
- deforestation

This leads to scatter plots where a large quantity of observations is concentrated in a single point or line.

But, copulas assume continuous, equally spread data instead.

Zero inflated data II



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copulas for zero inflated data - TMC I

An approach to solve this problem is by introducing *truly mixed copulas* (*TMC*) [Gräler et al. 2010].

The unit square is broken up into four areas: the lower left rectangle denoting the zero-zero pairs, the top left and lower right rectangles denoting the zero-non-zero and non-zero-zero pairs and the top right corner which can be rescaled and modeled as copula.

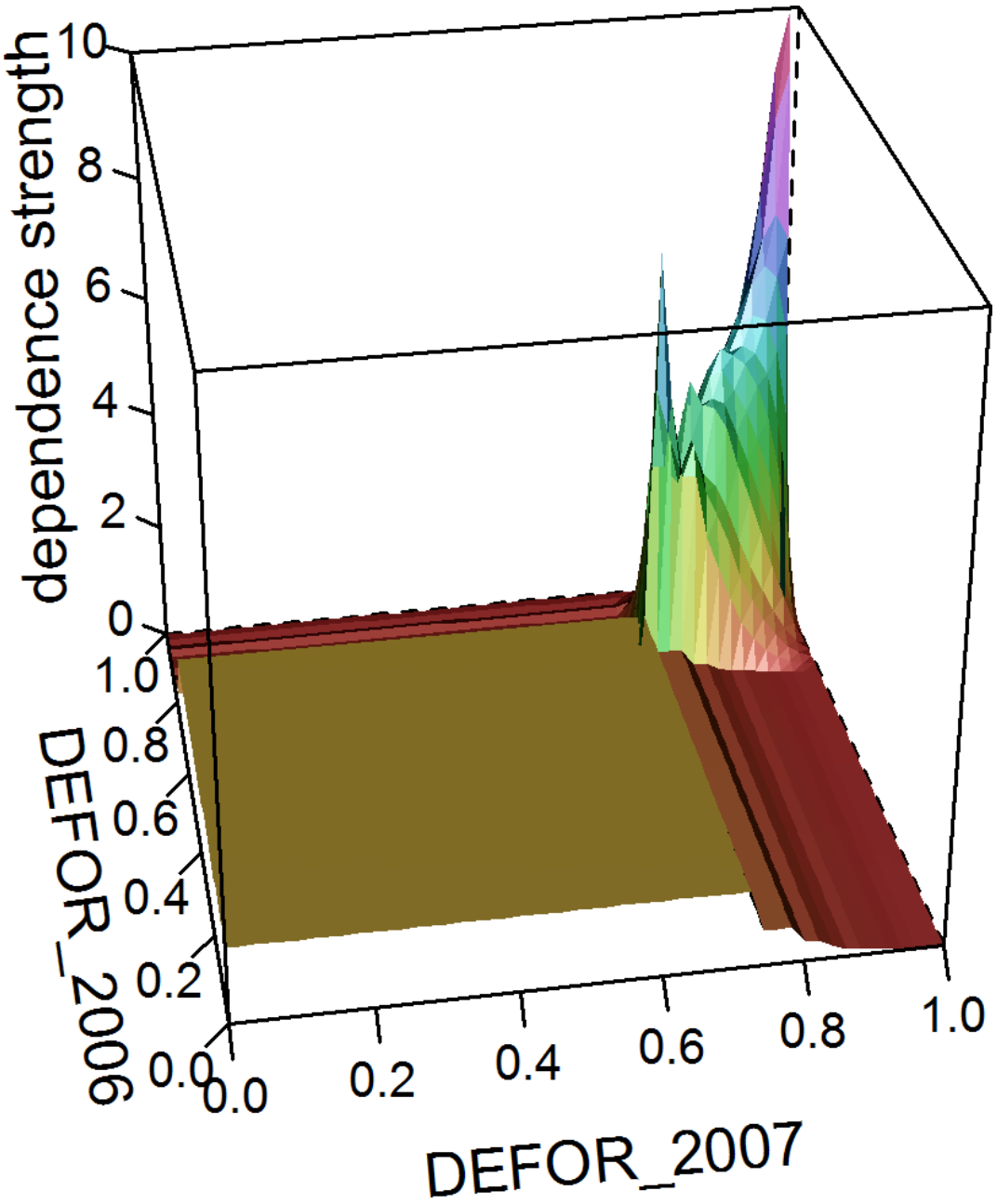
copulas for zero inflated data - TMC II

The rescaling is done in a way that the joint bivariate function is a copula again maintaining the mass relations and copula properties:

To achieve this, we need to estimate inner marginal functions and counter parts such that both add up to a constant 1.

A truly mixed copula density might look like:

copulas for zero inflated data - TMC III



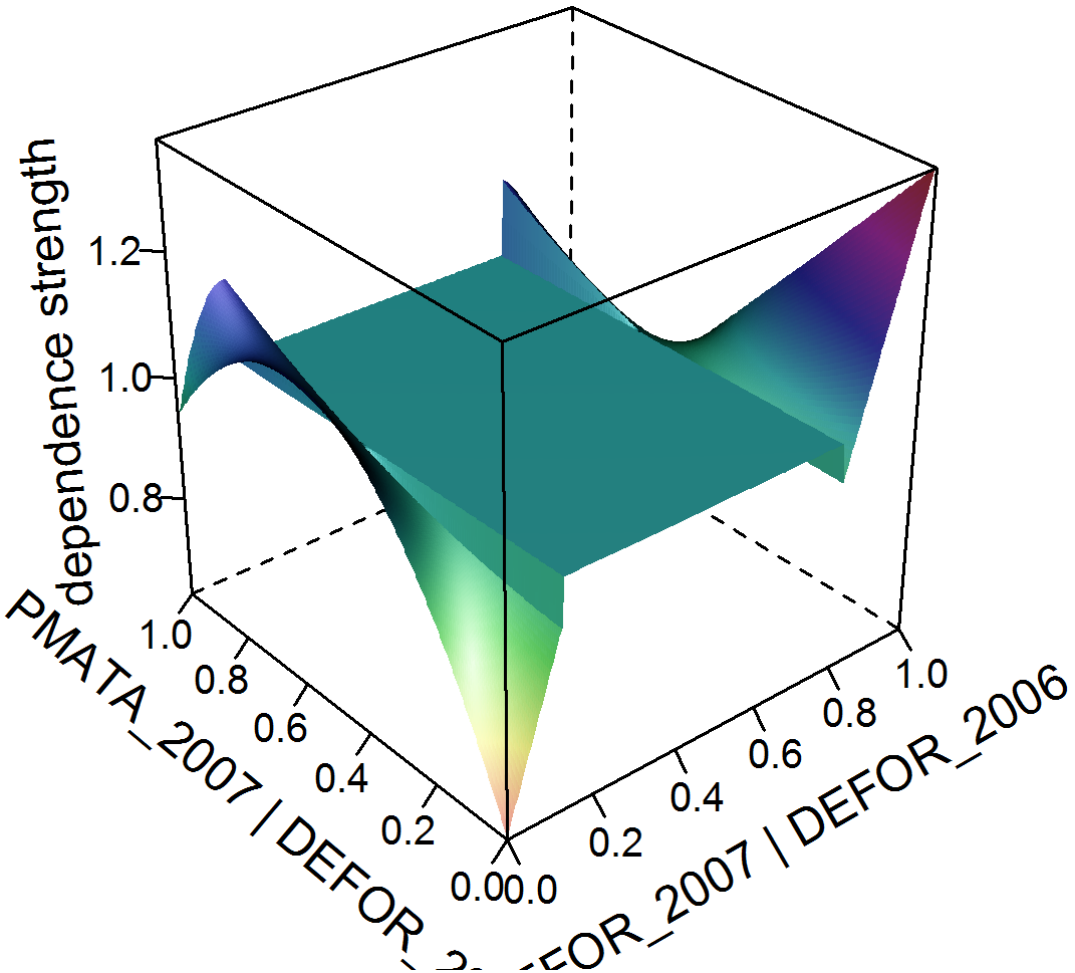
cutted copulas I

Instead of in one corner one might find a big bunch of values some where in the middle of the unit interval.

This part can be cut out according to its mass and inserted after the estimation process [Gräler et al. 2010].

Depending on the distribution of this cut-out, a distribution function might be necessary. The cut copula looks like

cutted copulas II



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Deforestation of the Amazon

The following study is published in [Gräler et al. 2010] and was presented at the Research Symposium GIScience for Environmental Change, November 27, 2010, Campos do Jordão (São Paulo), Brazil.

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Deforestation in the Amazon

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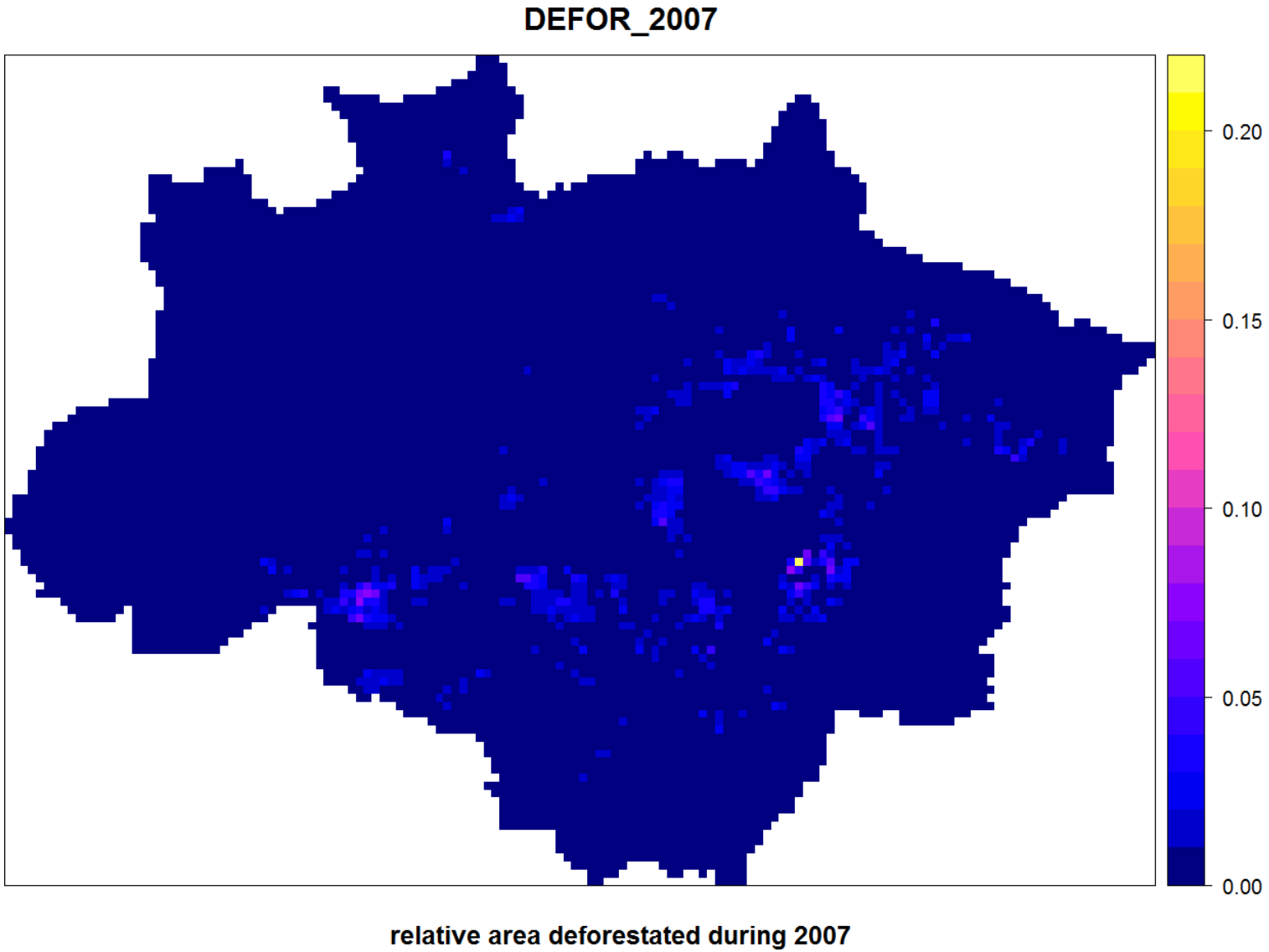
The data I

The amount of yearly deforested area per raster cell is calculated by INPE, Brazil.



The data II

Additional variables present are e.g.

- demographic information
- altitude
- preserved ares
- price of forest land
- area of sugarcane or soy beans

and many more!

We will investigate the three dimensional random process given by:

$$\text{defores. } 2006 \approx \text{defores. } 2007 \approx \text{price of forest } 2007$$

zero inflated data

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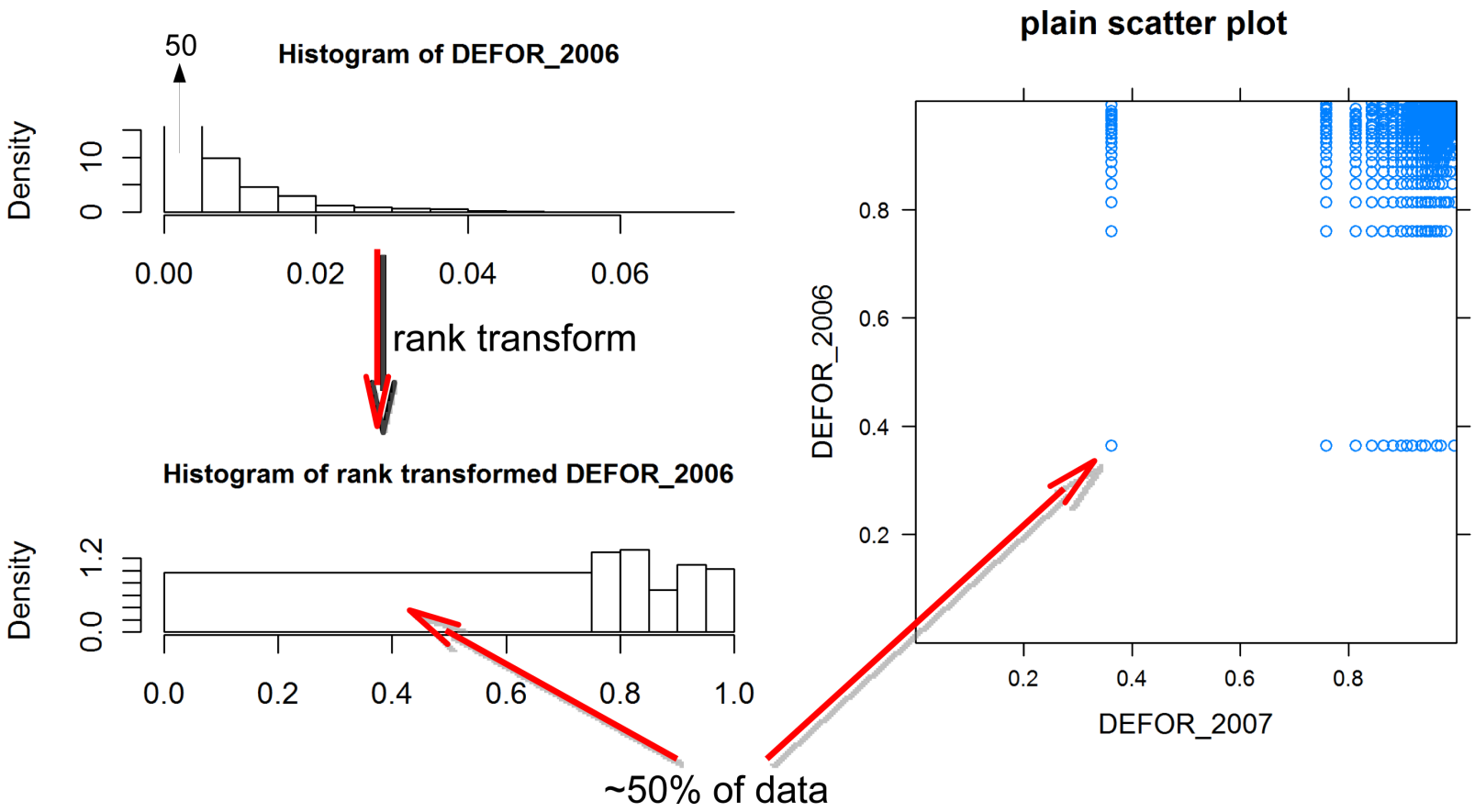
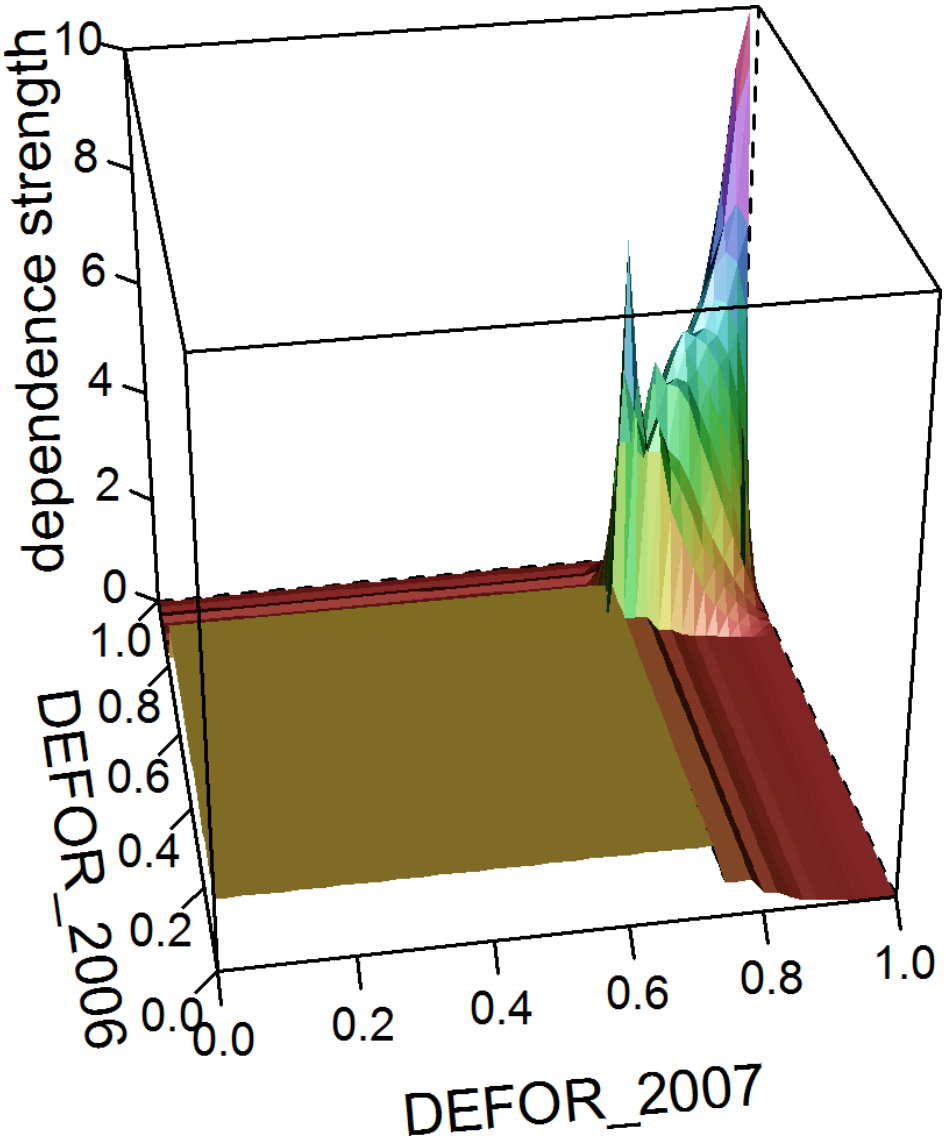
About 50% of the data is 0:



truly mixed copulas

In order to cope with this massive amounts of 0s we cut the copula in four parts: both margins are 0, either one is 0 and none of both is 0.



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fitting the TMC

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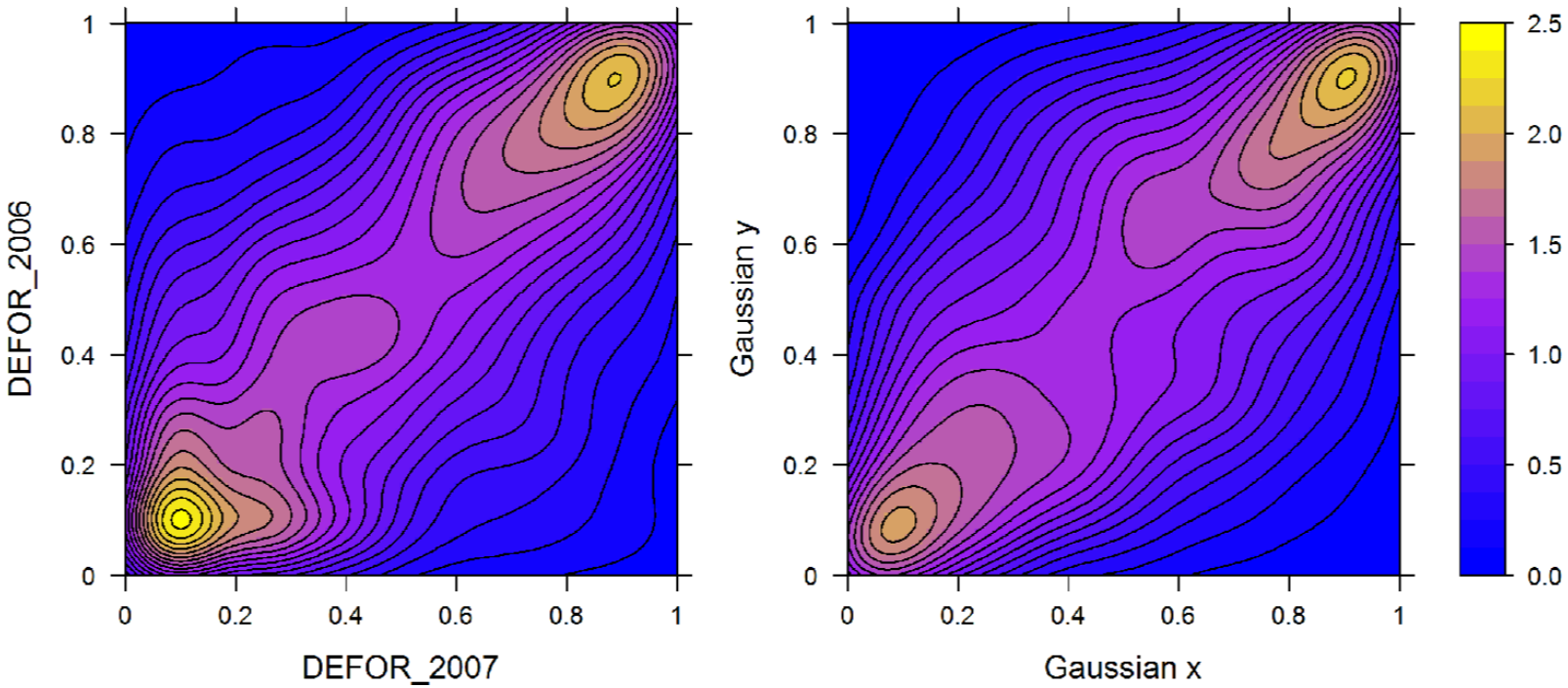
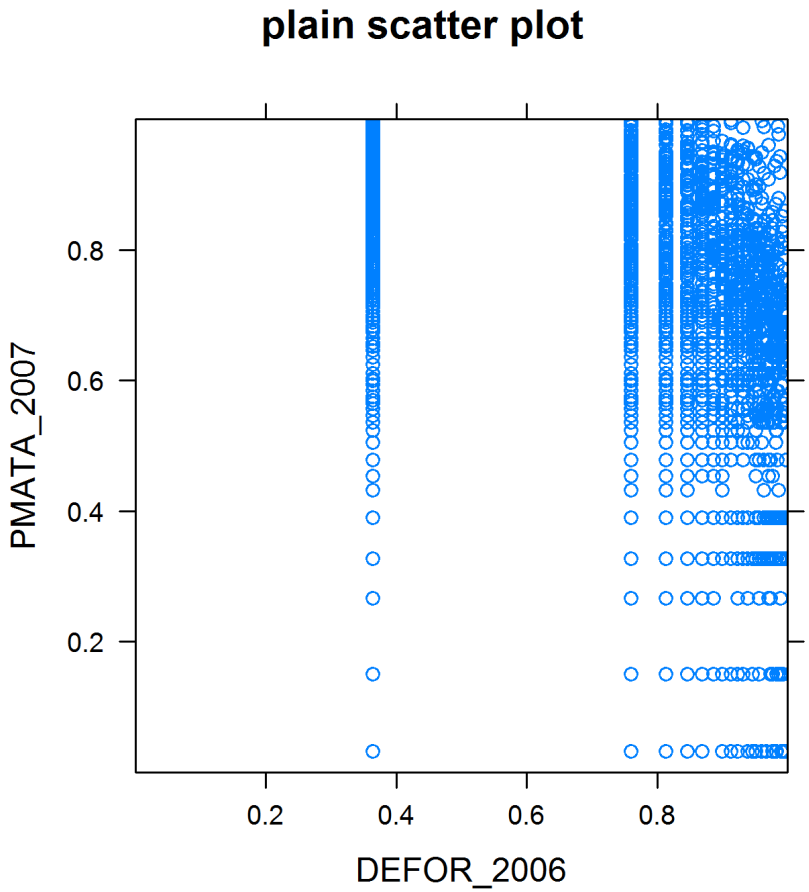
The top right part is rescaled to uniformity and a copula is fitted:



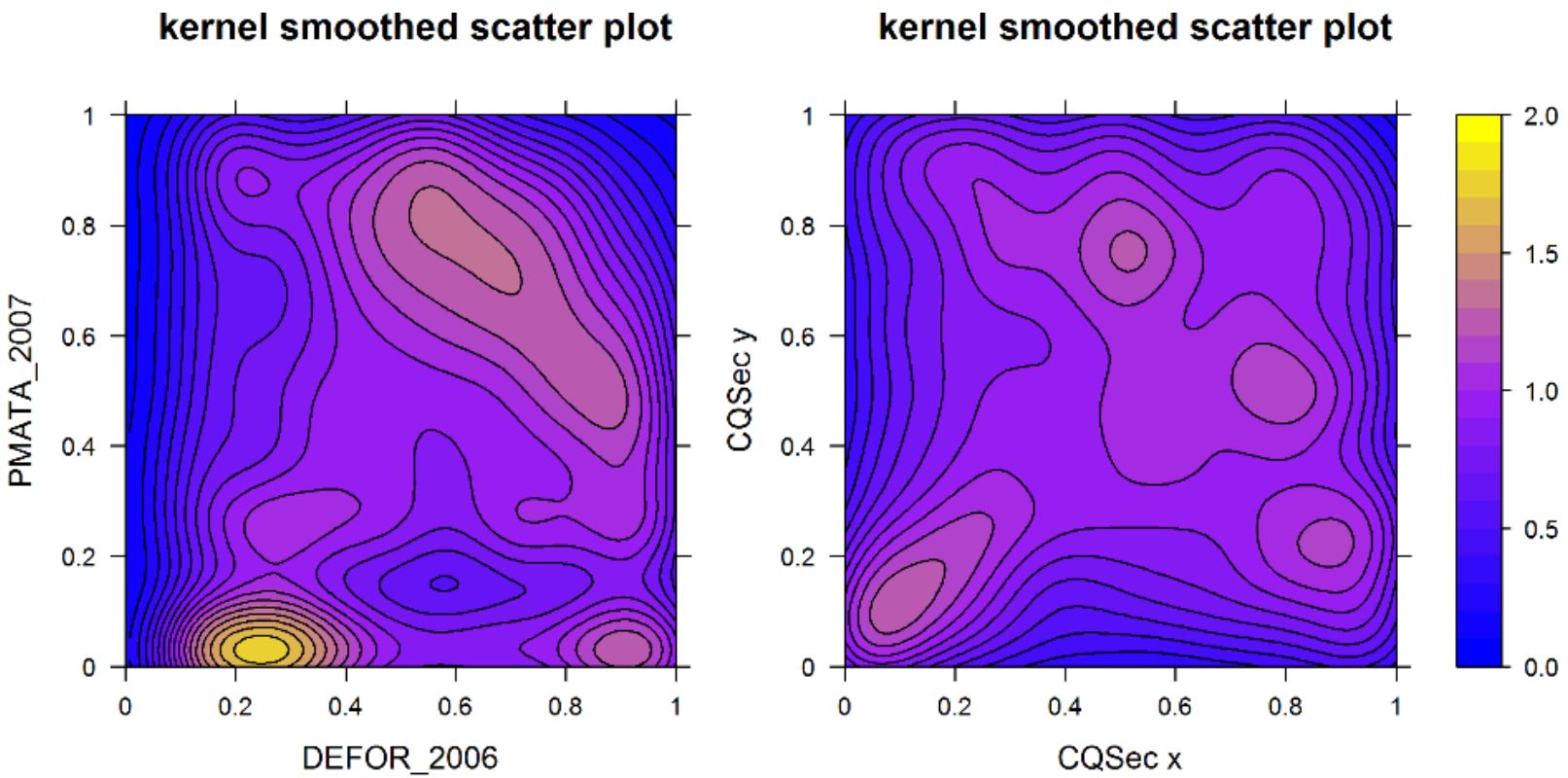
The three other parts are modeled through empirical cumulative distribution functions.

the second pair I

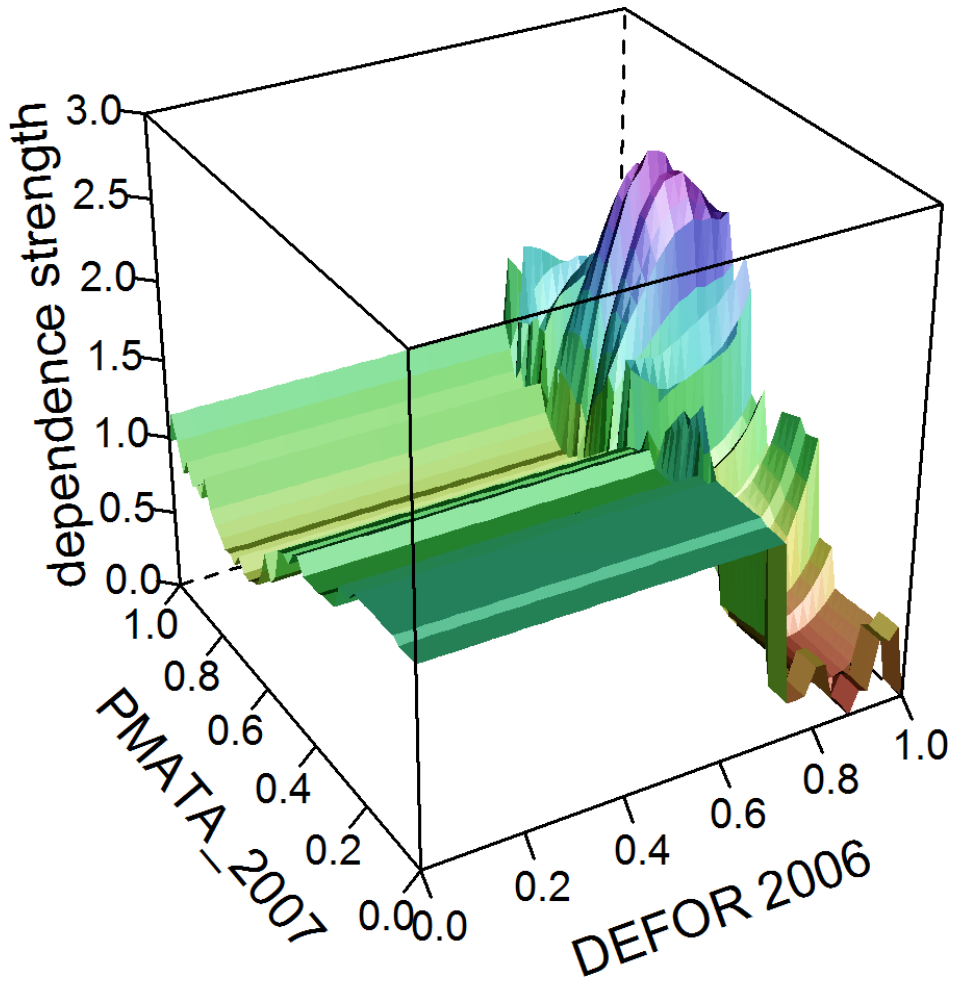
The same procedure is applied to the second pair variables:



the second pair II

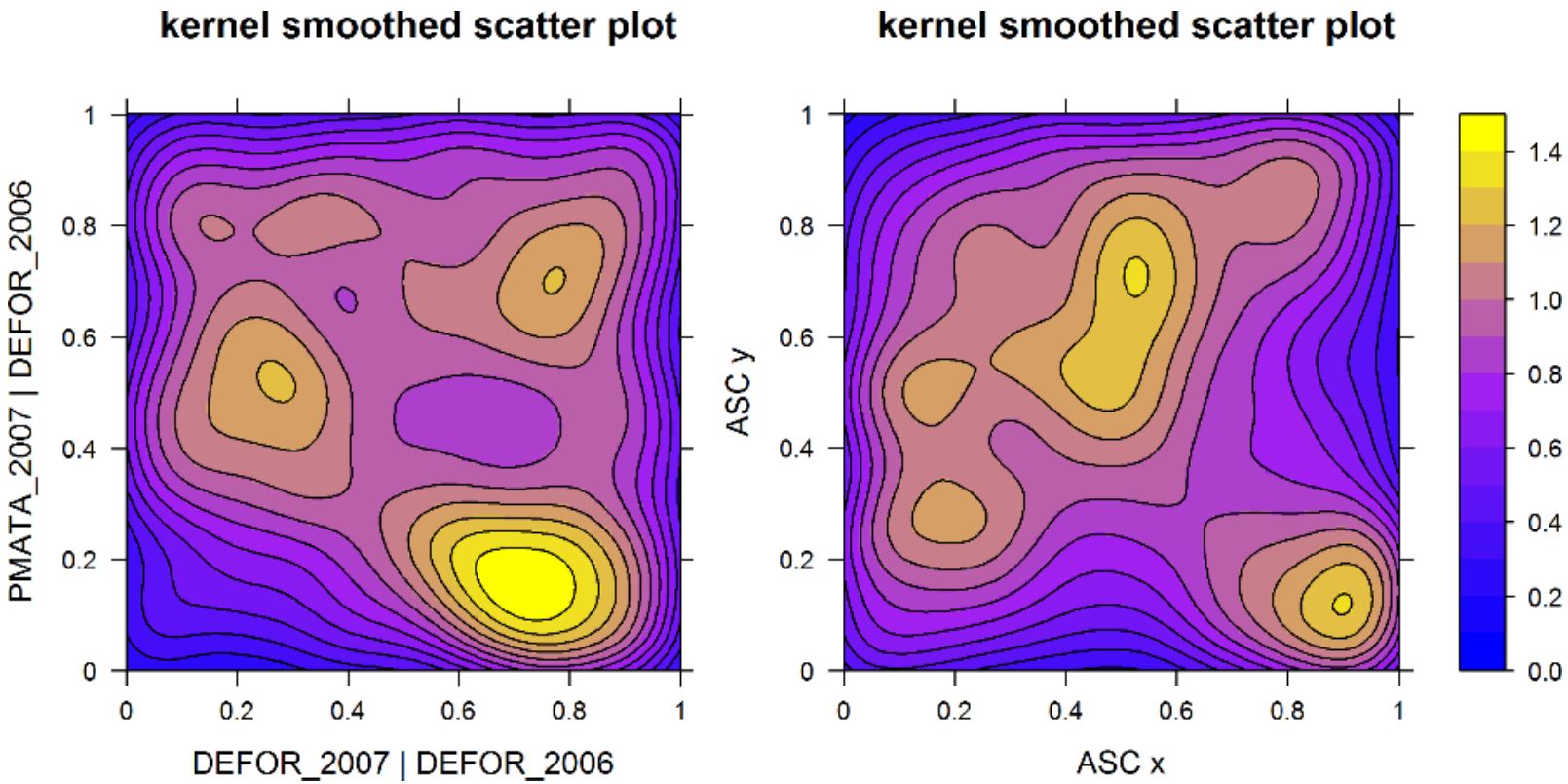


the second pair III

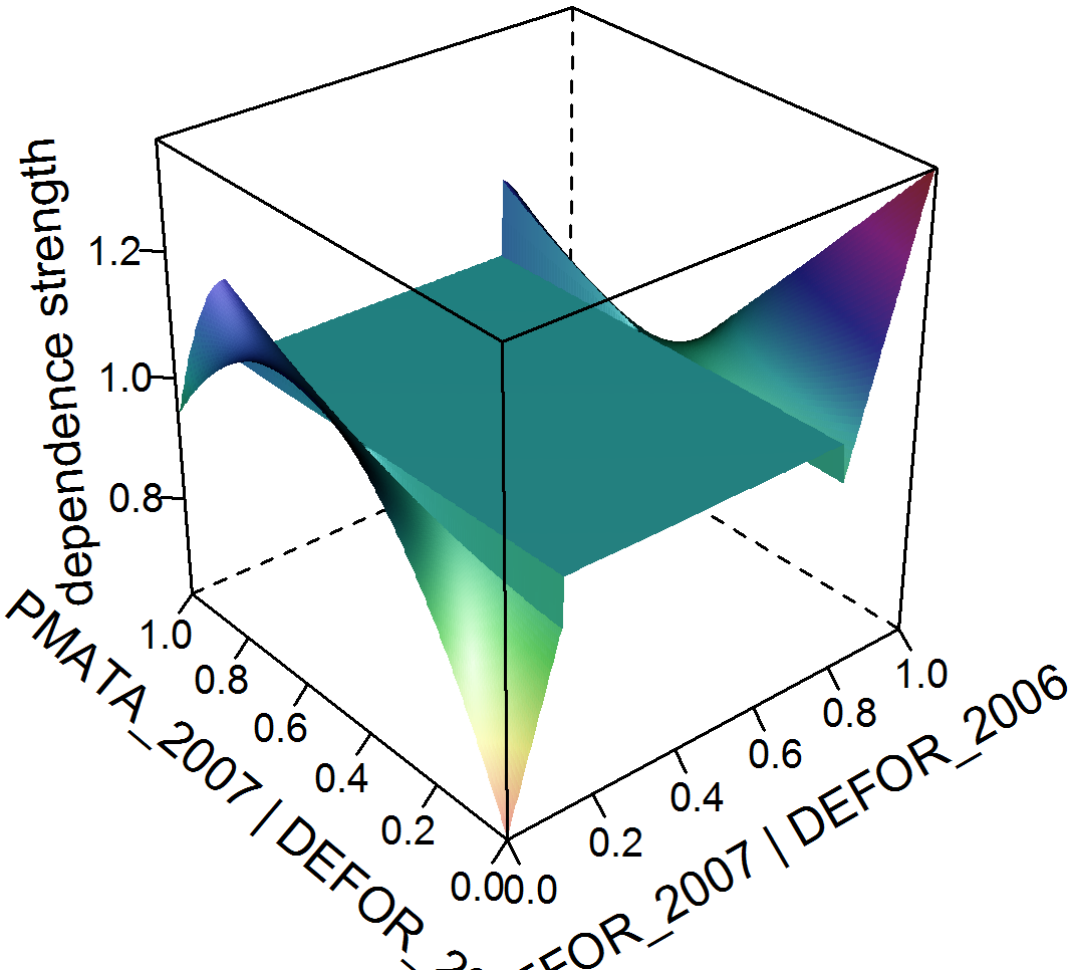


the cutted copula I

After the transformation of the data under the conditional distribution theres is a second value which takes a massive mass.
The remaining copula is:



the cutted copula II



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the joined copula

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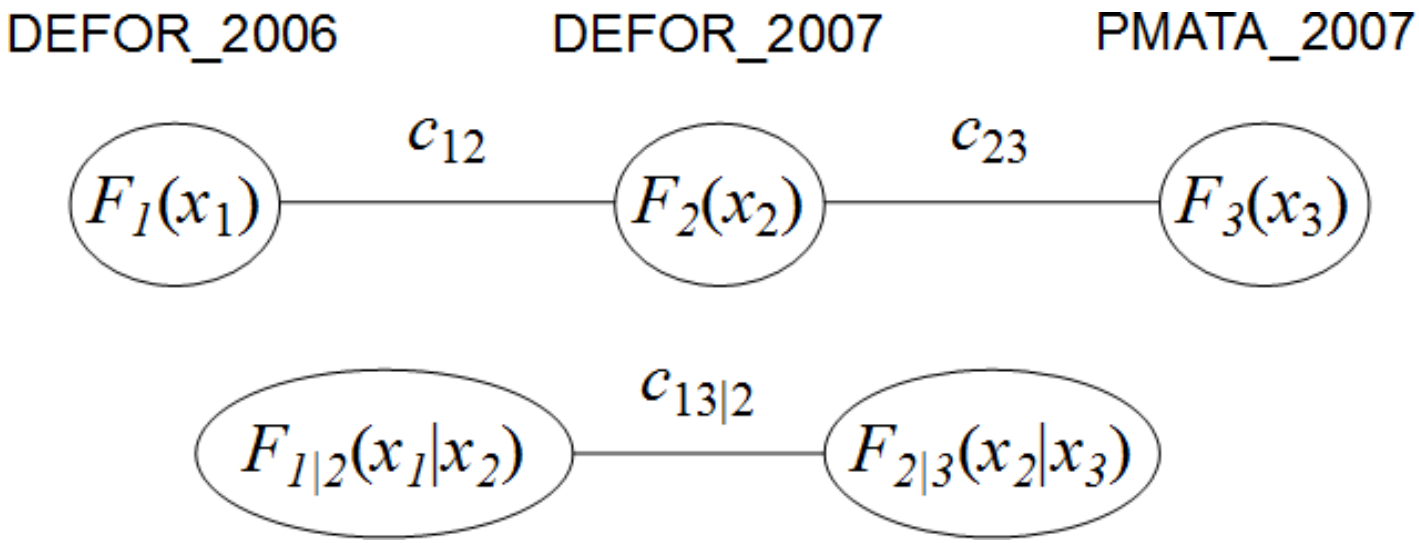
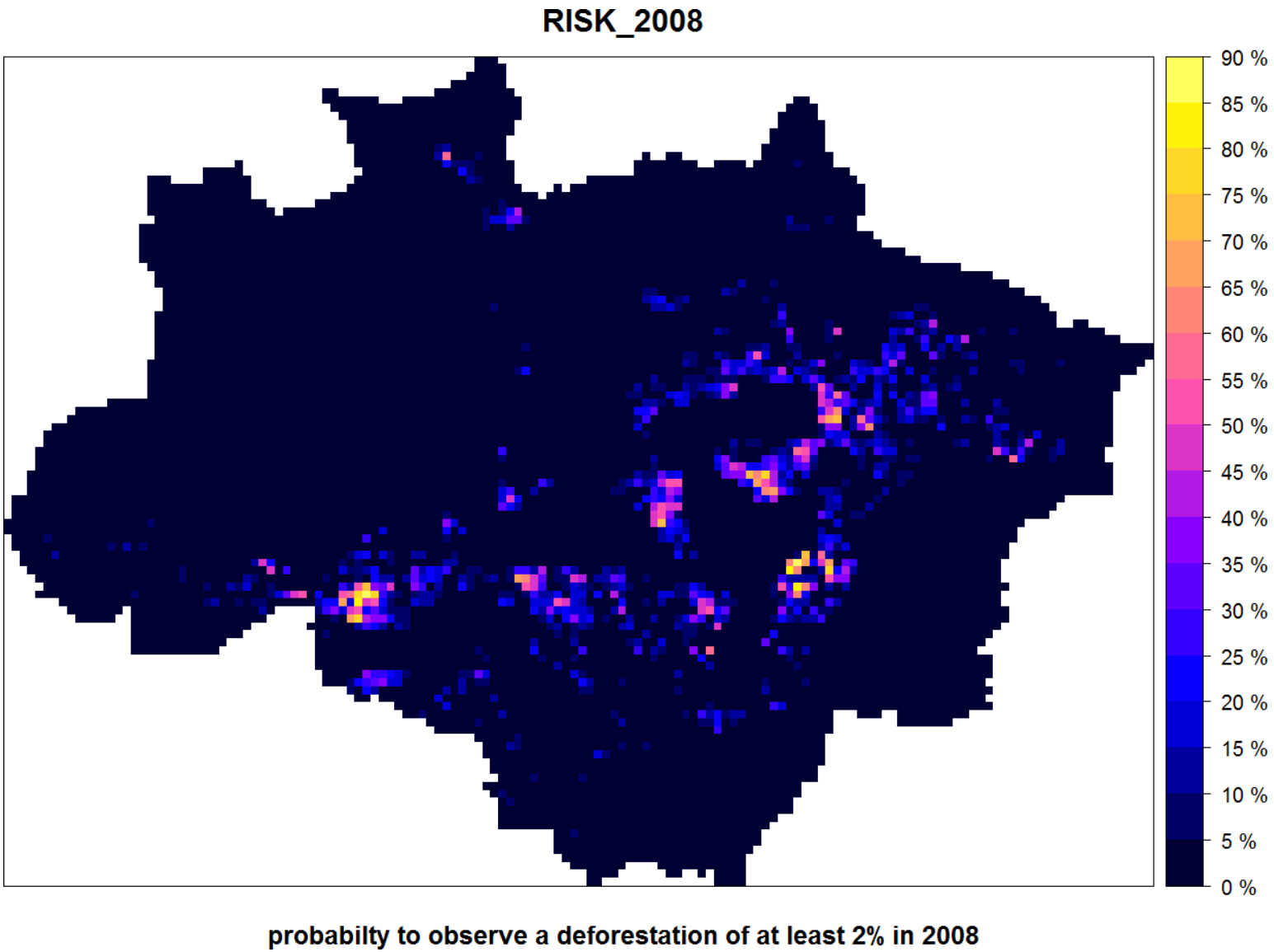
After we estimated the three pieces we can put them together:



Risk

Assuming temporal stationarity lets us calculate a risk map of deforestation for a given threshold



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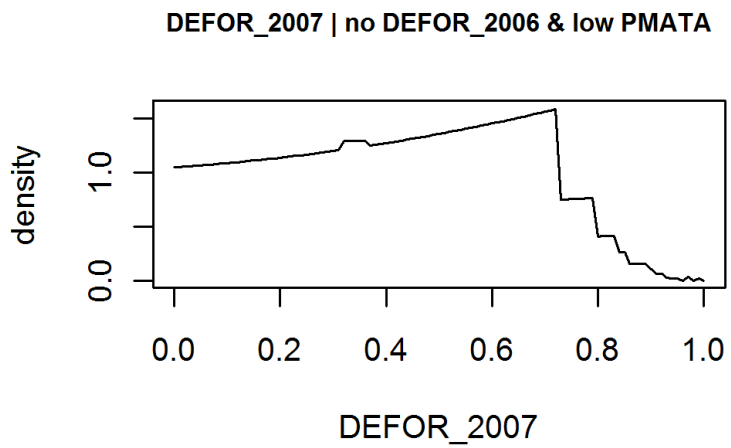
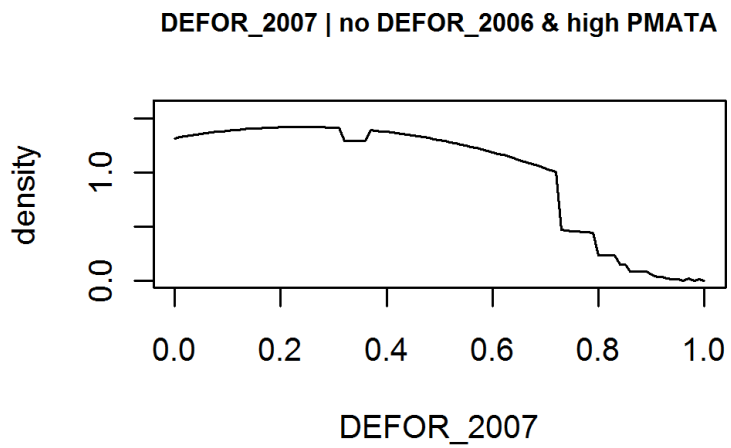
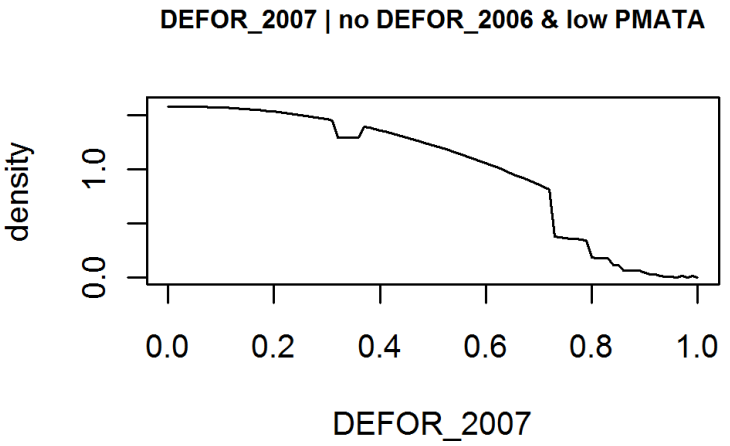
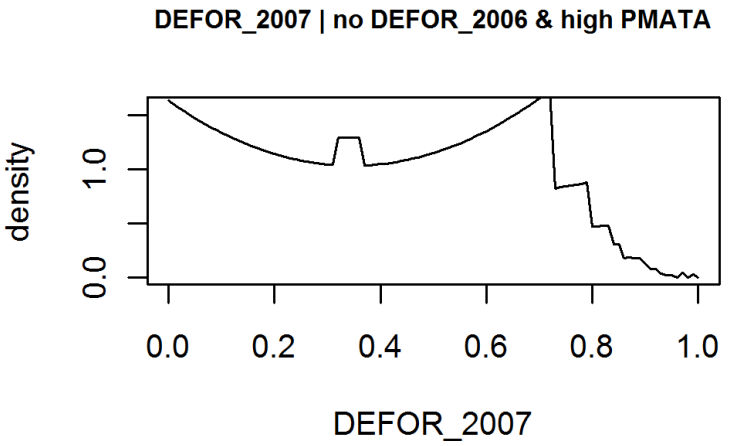
inspecting conditional densities for different copulas

Substituting the CQSec copula (top row) C_{23} with the best Gaussian (bottom row) has a visible impact:









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Practical

Choose your own triple of variables of the amazon / meuse / your data and try to estimate a pair copula.

or

Continue with the local neighborhood approach from the last practical to design an interpolation method incorporating the conditional density of the copula.

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References & further readings I

- Gräler, Benedikt (2009), 'Copulas for Spatio-Temporal Random Fields', Diploma thesis at the Institute of Mathematical Statistics and Institue for Geoinformatics, University of Muenster.
- Gräler, B., H. Kazianka & G. M. de Espindola (2010): "Copulas, a novel approach to model spatial and spatio-temporal dependence". In K. Henneböhl, L. Vinhas, E. Pebesma, & G. Cãmara (Eds.), GIScience for Environmental Change Symposium Proceedings, ifgiprints (Vol. 40, pp. 49-54). Presented at the GIScience for Environmental Change, November 27, 2010, Campos do Jordão (São Paulo), Brazil: AKA Verlag.

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References & further readings II

- Gräler, Benedikt & Edzer Pebesma (2011): The pair-copula construction for spatial data: a new approach to model spatial dependency. Poster at: Spatial Statistics 2011 - Mapping global change. Enschede, The Netherlands, 23-25 March 2011 (accepted for prersentation and publications in *Procedia Environmental Sciences* by Elsevier).
- Nelsen R. B. (2006): An Introduction to Copulas, 2nd Edition, Springer Science+Buisness, New York